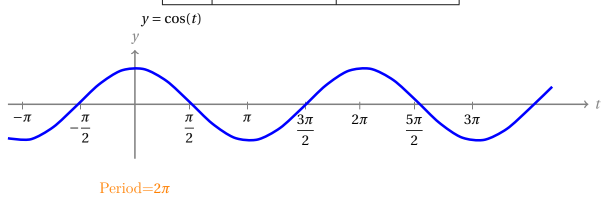
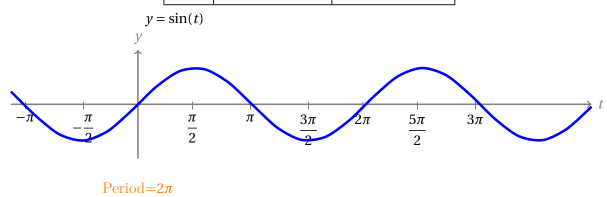
6.1: Graphs of Sine and Cosine

Using the points on the unit circle, we graph the sine and the cosine of t. For example, the points in the first quadrants will be as following.

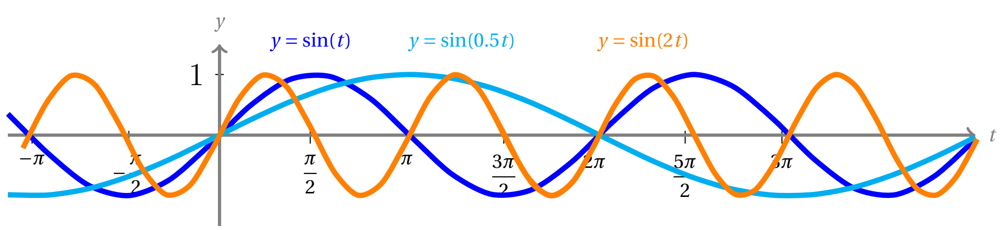
t	$\cos(t)$	point
0	1	$(0,1)$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\left(\frac{\pi}{6},\frac{\sqrt{3}}{2}\right)$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\left(\frac{\pi}{4},\frac{\sqrt{2}}{2}\right)$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\left(\frac{\pi}{3},\frac{1}{2}\right)$
$\frac{\pi}{2}$	0	$\left(\frac{\pi}{2},0\right)$

t	$\sin(t)$	point
0	0	$(0,0)$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\left(\frac{\pi}{6},\frac{1}{2}\right)$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\left(\frac{\pi}{4},\frac{\sqrt{2}}{2}\right)$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\left(\frac{\pi}{3},\frac{\sqrt{3}}{2}\right)$
$\frac{\pi}{2}$	1	$\left(\frac{\pi}{2},1\right)$



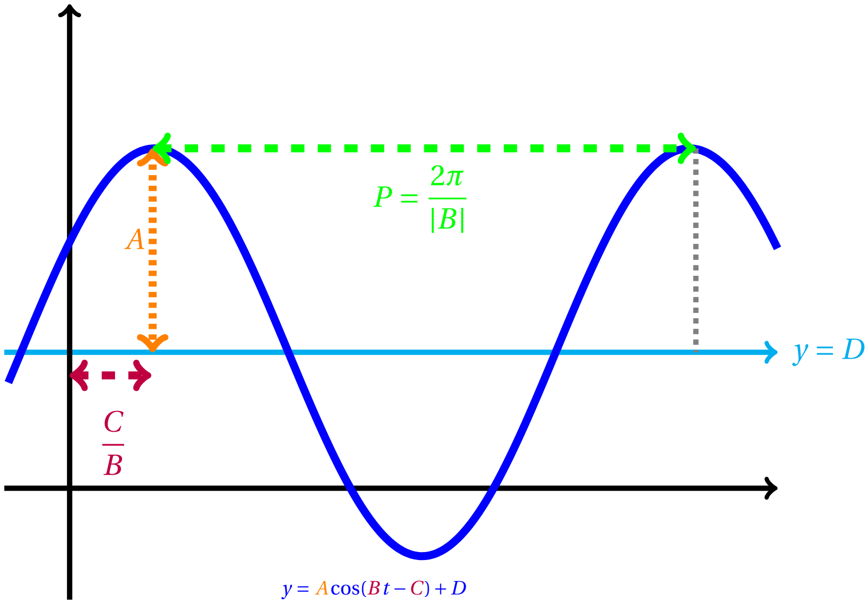


- Comparing sine functions:



- A sinusoidal function any function that can be expressed in the form $f(t) = A\sin(Bt - C) + D$ or $f(t) = A\cos(Bt - C) + D$.
- Midline: The horizontal line $y = D$, where D appears in the general form of a sinusoidal function. (It is called midline because D is the average y-value.)
- Amplitude:: The greatest vertical distance of a function from the midline; the absolute value of the constant A appearing in the definition of a sinusoidal function.
- A periodic function: A function $f(t)$ that satisfies $f(t+P) = f(t)$ for a specific constant P and any value of t. (P is the smallest positive value that satisfies such equation and is called the period.) The formula $P = \dfrac{2\pi}{|B|}$ gives the period.

- Phase shift The horizontal displacement of the basic sine or cosine function; the constant $\frac{C}{B}$ for $-2\pi < C < 2\pi$.



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  Now, you can complete Problems 1-3.

Transformations:

- The above picture can be explained using transformations, but using the above information is recommended instead.
- **How to graph:** Find the local max and min points, amplitude, period, phase shift and midline, then graph.

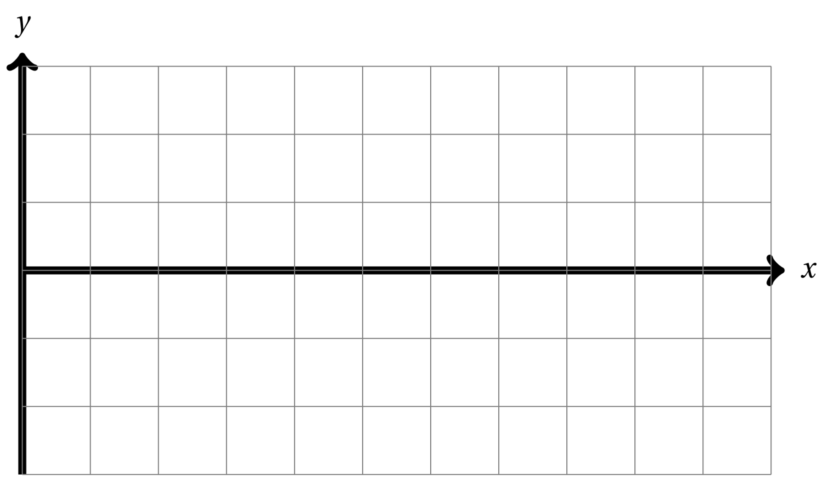
  Now, you can complete Problem 4.

1. The period of $f(x) = 2\cos(4x + \pi/6)$ is

   (a) 2π (b) $\frac{\pi}{2}$ (c) $\pi/2$ (d) 4π

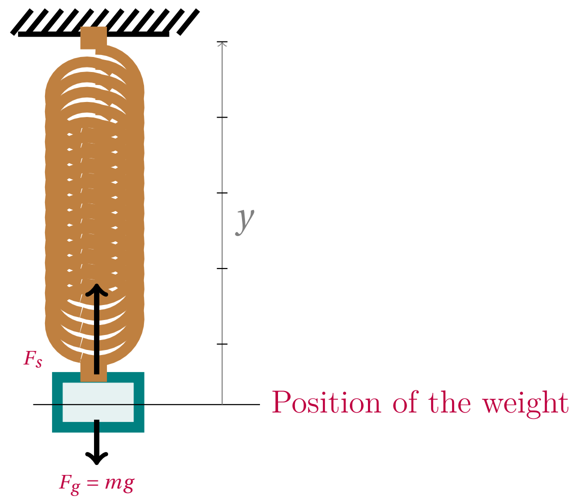
2. Find a function that models the simple harmonic motion having **Period** 4 and **amplitude** 10. Assume that the **initial displacement is zero**, at time $t = 0$.

3. Sketch two periods of the graph of $y = \frac{1}{3}\sin(2x - \frac{\pi}{2})$, labeling the **maximum** and **minimum** height, the x-intercepts and two more points on one period. List the **amplitude**, **period** and **phase shift** of $f(x)$.



4. **Mechanical Engineering:**

A weight is attached to a spring that is then hung from a board, as shown in the figure. As the spring oscillates up and down, the position (y) of the weight relative to the board ranges from -2 in. (at time $t = 0$ second) to -6 in. (at time $t = 2\pi$ second) below the board. Assume the position (y) is given as a sinusoidal function of (t). Motion of this spring mass system is a simple harmonic motion.



https://www.geogebra.org/m/ygcvqa9m

(A) Find **amplitude**, **period** and **vertical shift** (gives midline).

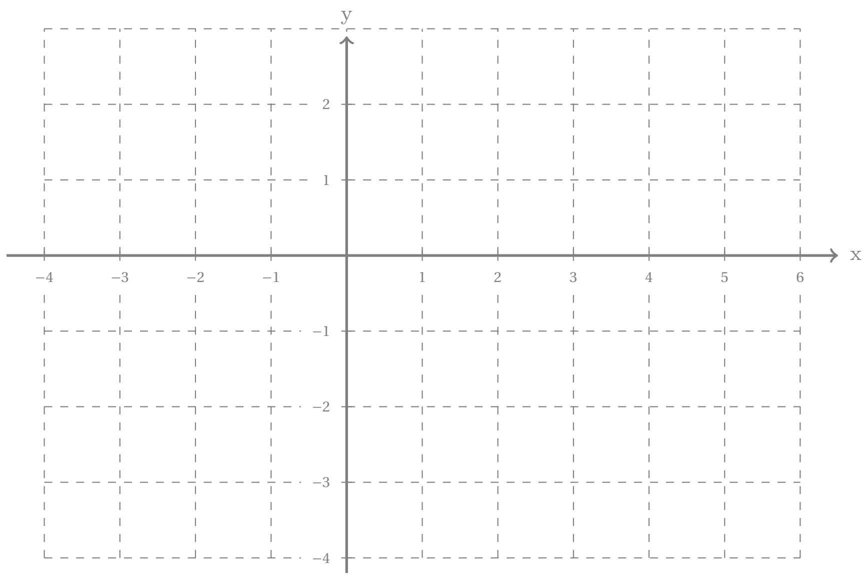
(B) Find a cosine function that gives the position (y) in terms of (t).

(C) Sketch a graph of the function for at least two periods; noting Part (A).

(D) Find a sine function for this motion and graph it.

5. *Optional:* Graph

$$f(x) = \begin{cases} x & x < -2 \\ \sin(x) & -2 \leq x \leq 0 \\ -x^2 & 0 < x < 2 \\ \cos(x) & x \geq 2 \end{cases} .$$



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